On Block-Reference Coherent Diffraction Imaging (CDI)

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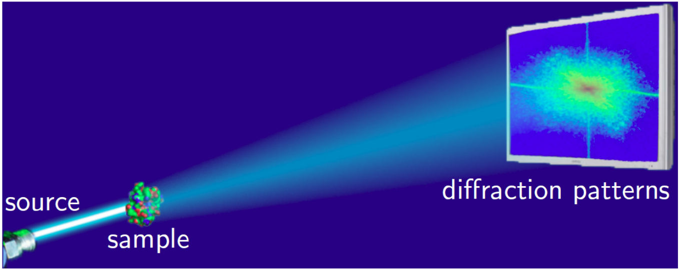
SLAC

CDI and phase retrieval

Detectors record intensities of diffracted rays $\rightarrow$ phaseless data only!

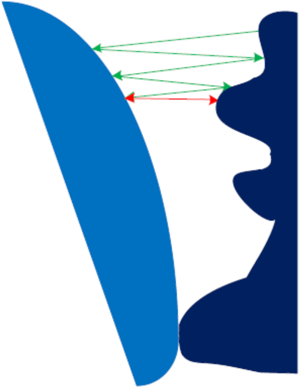


Fraunhofer diffraction $\longrightarrow$ intensity of electrical field $\approx$ Fourier transform

$$|\hat{x}(f_1, f_2)|^2 = \left| \int x(t_1, t_2) e^{-i2\pi(f_1 t_1 + f_2 t_2)} \, dt_1 dt_2 \right|^2$$

Phase retrieval algorithms

- Standard approach: Alternating projections method
- E.g. Fienup's Hybrid Input-Output (HIO) Method, etc.
- No guaranteed convergence (projection onto nonconvex sets)
- Often slow in practice



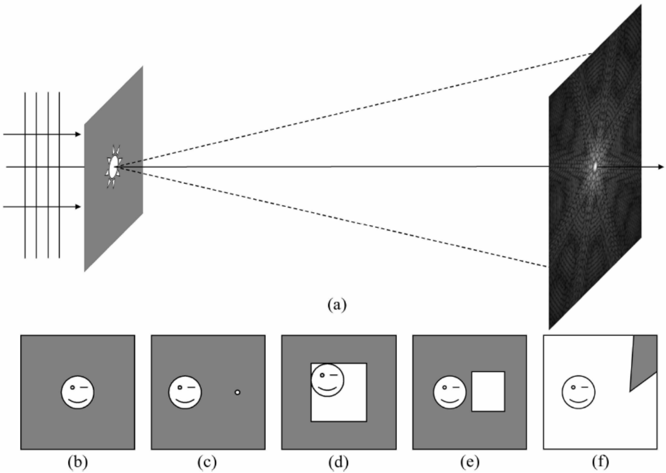


Figure: Fienup et. al. HERALDO (2007)

Extended-reference imaging

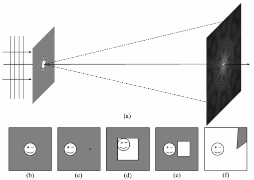


Figure: Fienup et. al. HERALDO (2007)

- Add an adjacent reference
- Guaranteed recovery
- Solve a linear system

Prior art

- Various proposed extended reference schemes:
  - Fourier holography
  - Podorov and Paganin (2007)
  - Guizar-Sicairos and Fienup (2007)
- Algorithms presented are reference-specific

Our work

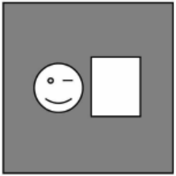
- Unified viewpoint for solving with a generic reference

- Analytical noise stability analysis
  - $\Rightarrow$ Explains performance of different references
  - $\Rightarrow$ More supporting evidence for a block-reference

Fourier duality

- Recall: multiplication $\underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\leftrightharpoons}}$ convolution
- $|\mathcal{F}(x)|^2 \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\leftrightharpoons}} x \star x$

Convolution



$$
\begin{array}{ccccccccccccccc}
* & * & * & * & a & b & c & d & & & & & & & \\
* & * & * & * & e & f & g & h & & & & & & & \\
* & * & * & * & i & j & k & l & & & & & & & \\
* & * & * & * & m & n & o & \color{blue}{*} & * & * & * & a & b & c & d \\
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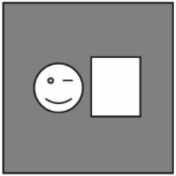

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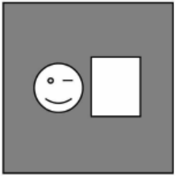

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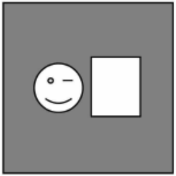
Convolution



*	*	*	*	a	b	c	*	*	*	*	a	b	c	d
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*	*	*	*	i	j	k	*	*	*	*	i	j	k	l
*	*	*	*	m	n	o	*	*	*	*	m	n	o	p

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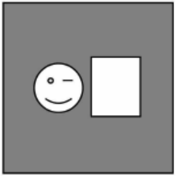

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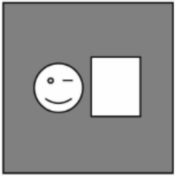
Convolution



*	*	*	*	*a*	*b*	*c*	*d*						
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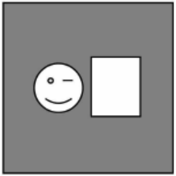
*	*	*	*	*i*	*j*	*	*	*	*	*a*	*b*	*c*	*d*
*	*	*	*	*m*	*n*	*	*	*	*	*e*	*f*	*g*	*h*
						*	*	*	*	*i*	*j*	*k*	*l*
						*	*	*	*	*m*	*n*	*o*	*p*

Convolution



$*$	$*$	$*$	$*$	a	b	c	d						
$*$	$*$	$*$	$*$	e	f	$*$	$*$	$*$	$*$	a	b	c	d
$*$	$*$	$*$	$*$	i	j	$*$	$*$	$*$	$*$	e	f	g	h
$*$	$*$	$*$	$*$	m	n	$*$	$*$	$*$	$*$	i	j	k	l
						$*$	$*$	$*$	$*$	m	n	o	p

Generic algorithm

Given $[X, R]$, solve:

$$\min_X \frac{1}{2} \| \widehat{C}_{[R,X]} - (R \star X)_Q \|_2^2$$

- $\widehat{C}_{[R,X]}$ is the (top-left) cross-correlation data
- $(R \star X)_Q$ is the (top-left) cross-correlation operator on X

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Note: $(R \star X)_Q$ is a linear operator!

$\Rightarrow \widehat{X} = M_R^{-1} \left(\widehat{C}_{[R,X]} \right)$.

Some algorithm analysis

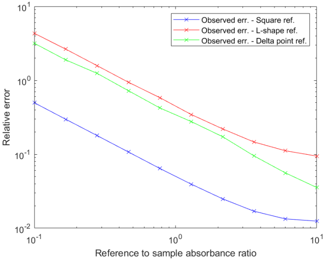
- Subsumes reference-specific algorithms
  - Point reference (Fourier holography)
  - L-shape reference (Fienup)
  - Block reference (Podorov)

- M_R is lower-triangular $\Rightarrow O(n^2)$ runtime, $O(n)$ in special cases

- Robust to noise and generalizable to beamstop

Analytical noise stability analysis

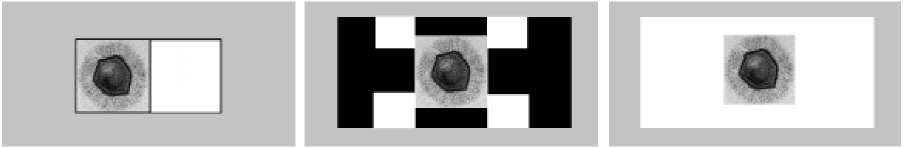
- Under Poisson noise, can explicitly calculate $\mathbb{E}\|\hat{X} - X\|^2$
  - $\Rightarrow$ For low-frequency images, block reference has lowest error!
- Comparison of holography, L-shape, and block references



  - $\Rightarrow$ Error decreases with ref. absorption

Block-reference

- Constant absorption density
- Incident area greater or equal to specimen



Fabrication

- Possible via lithography and nano-scale printing.
- Prototype design in process at SLAC.